6
Unfolding

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An unfolding problem with two measured values

The response matrix $\mathbf{A}$ is a 2-by-2 matrix and is easily inverted:

$$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \qquad \mathbf{A}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

Unfolding with the matrix method is done according to the equation $\mathbf{A}^{-1}\mathbf{y} = \widehat{\mathbf{x}}$, or

$$\frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \widehat{x}_1 \\ \widehat{x}_2 \end{pmatrix} = \frac{1}{ad-bc} \begin{pmatrix} d\, y_1 - b\, y_2 \\ -c\, y_1 + a\, y_2 \end{pmatrix}$$

in a single step. The two components y_1 and y_2 of vector $\mathbf{y}$ are independent, their variance is assumed to be equal to y_1 and y_2 (Poisson statistics). The covariance matrix of the unfolded values is (by error propagation)

$$\mathbf{V}_x = \mathbf{A}^{-1}\mathbf{V}_x \left(\mathbf{A}^{-1}\right)^T = \left(\frac{1}{ad-bc}\right)^2 \begin{pmatrix} d^2\, y_1 + b^2\, y_2 & -cd\, y_1 - ab\, y_2 \\ -cd\, y_1 - ab\, y_2 & c^2\, y_1 + a^2\, y_2 \end{pmatrix}.$$

The results for the two response matrices $\mathbf{A}_a$ and $\mathbf{A}_b$ are:

$$\begin{array}{rcc@{\qquad}rcc} & \text{(a)} & & & \text{(b)} & \\ \widehat{x}_1 & = & 22.9 \pm 5.8\,, & \widehat{x}_1 & = & 13.3 \pm 10.7\,, \\ \widehat{x}_2 & = & 17.1 \pm 5.3\,, & \widehat{x}_2 & = & 26.7 \pm 11.4\,, \\ \rho & = & -0.35\,, & \rho & = & -0.84\,. \end{array}$$

The worse resolution of case (b) results in a large (negative) correlation, and the uncertainties are doubled. The χ^2 values for the distance from the true value, calculated from the full covariance matrix of the unfolded data including the correlation, are $\chi^2_a = 0.15$ and $\chi^2_b = 1.18$. From the known true vector $\mathbf{x}$ the expected measured values can be calculated by $\mathbf{A}_a\mathbf{x}$ and $\mathbf{A}_b\mathbf{x}$. The χ^2 values calculated from the difference of the measured and the expected measured values are identical to the χ^2 values above.