Monte Carlo and large angle gluon radiation

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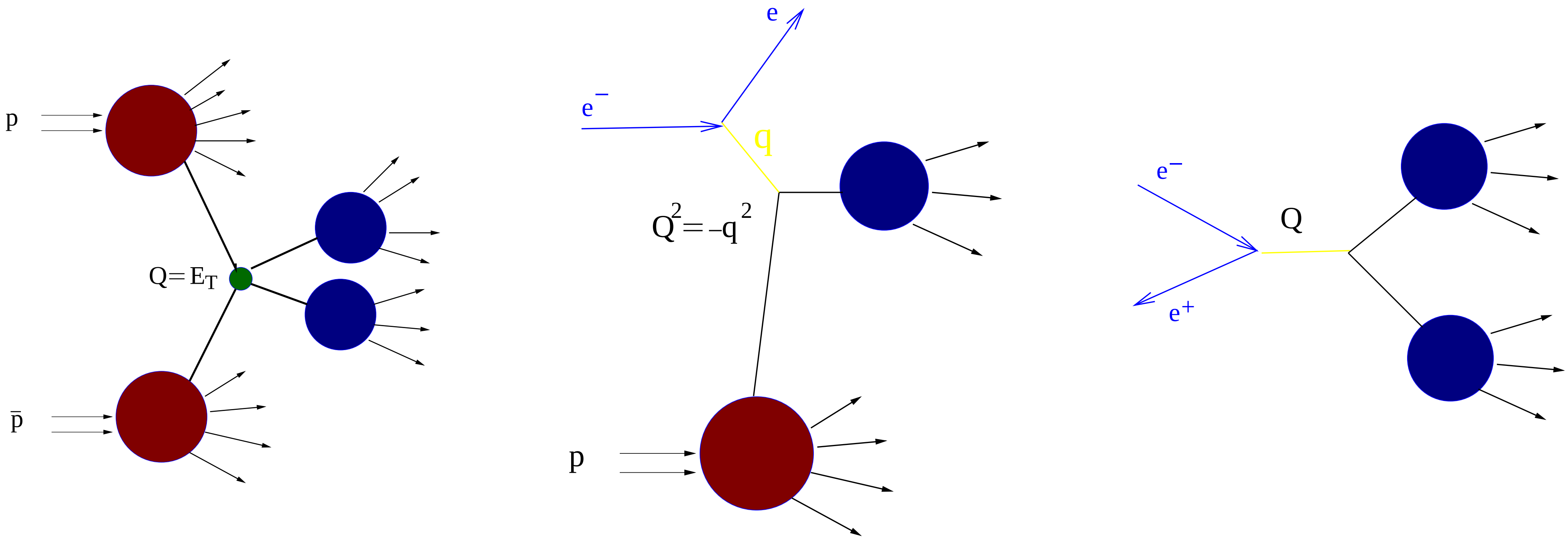
How is a MC made up: factorization structure of QCD



- hard $2 \to 2$ distribution at scale Q
- structure function
- fragmentation function

$p\bar{p} \to W^+ + X\,, \quad W^+ \to t\bar{b}:$ a MC event

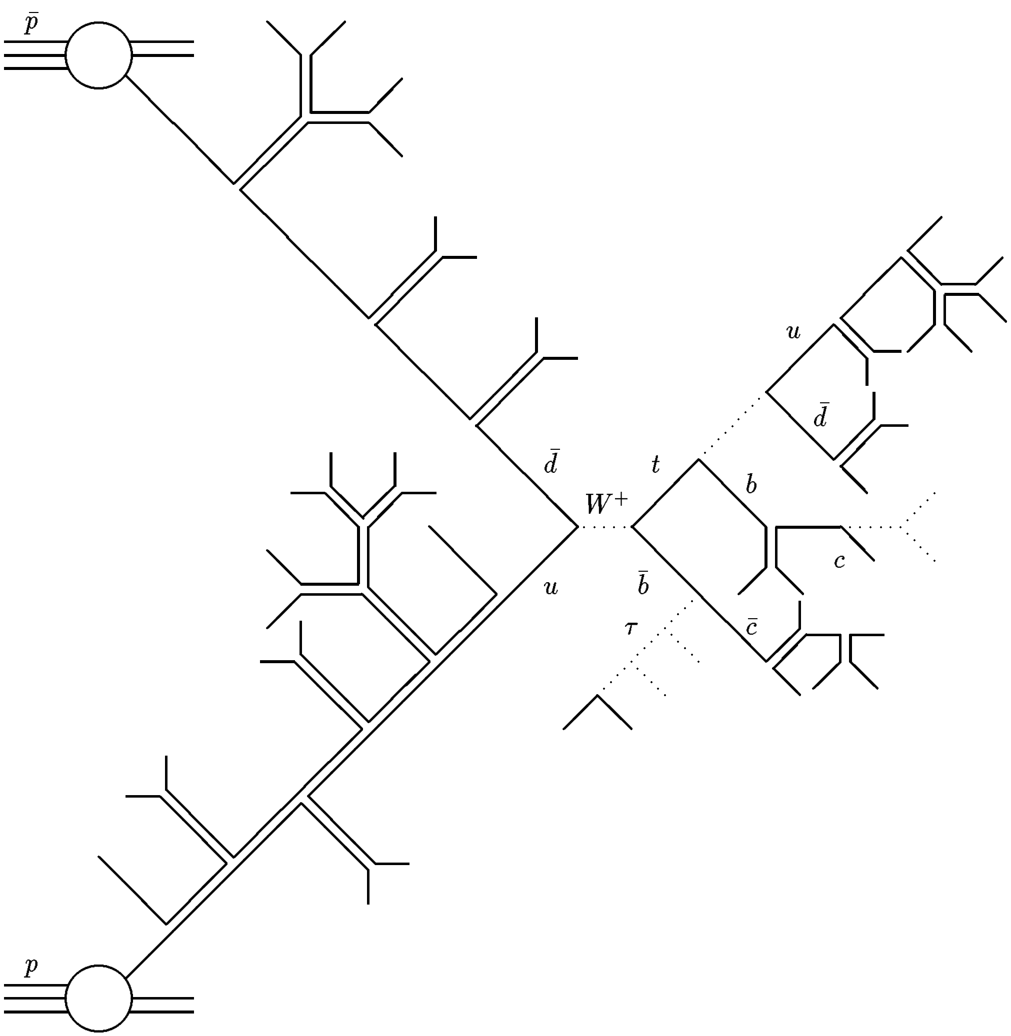


Figure 1: Colour structure of a $\bar{p}p \to W^+ + X$, $W^+ \to t\bar{b}$ event.

As a consequence of factorization, one can construct a single Monte Carlo program, such as

Theoretical tools (for Monte Carlo)

UV divergences: $\rightarrow$ running coupling $\alpha_s(Q)$

collinear divergences (2-parton collinear): $\rightarrow$ factorized logarithms

infrared divergences (soft gluon branching): $\rightarrow$ factorized logarithms

Resummation of (universal) collinear + IR factorized logarithms

Preconfinement (D.Amati, G.Veneziano, PL83(79)87)

1) colour singlet mass $\rightarrow$ Sudakov suppression

2) small colour singlet mass $\rightarrow$ model $\rightarrow$ hadrons

3) results not too sensitive to hadronization model

Structure function + fragmentation function enter e^+e^-, ℓ–h, h-h

Attempt to improve Monte Carlo

Beyond present MC based on collinear factorization (e.g. HERWIG)

Problem: incorporating recoil into dipole emission

L.Lönnblad, Comp.Phys.Com.71 (1992) 15

Z.Nagy and D.E.Soper, JHEP 0807 (2008) 025

M.Dinsdale, M.Ternick, S.Weinzierl, Phys.Rev.D76(2008)094003

W.T.Giele, D.A.Kosower and P.Z.Skands, Phys.Rev.D78(2008)014026

Results of present attempt:

- Non-leading contributions to multiplicities are correct
- $Simple$ recoil strategy conflicts with collinear factorization:

  DGLAP evolution is wrong

Monte Carlo: generating functional for a process

$$e^+e^- \to \gamma^* \to p_a\, p_b\, q_1 \cdots q_n \qquad (\omega_i \ll E_{a,b} = \tfrac{1}{2}Q = E)$$

Soft emission: p_a, p_b hard quarks and q_i soft gluons

$$G_{ab}(E,u) = \sum_n \int |M_{ab}(q_1 \cdots q_n)|^2 \cdot \prod_i \omega_i d\omega_i \frac{d\Omega_i}{4\pi} \theta(E-\omega_i) u(q_i)$$

with $u(q_i)$ sources for soft emission

Normalization $G_{ab}(E, u{=}1){=}1$

Tree level formula for soft emission: planar

Bassetto, Ciafaloni and GM Phys.Rept.100(1983)201
Parke and Taylor Phys.Rev.Lett.56(1986)2459

$$|M_{ab}^{\rm three}(q_1 \cdots q_n)|^2 = \frac{\bar{\alpha}_s^n}{n!} \sum_{\rm perm} W_{ab}(q_{i_1} \cdots q_{i_n}) \qquad \bar{\alpha}_s = \frac{N_c \, \alpha_s}{\pi}$$

$$W_{ab}(q_1 \cdots q_n) = \frac{(ab)}{(aq_1) \cdots (q_n b)}$$

Strong ordering in c.m. $\omega_{i_n} \ll \cdots \ll \omega_{i1} \ll E$

Planar approximation: $N_c \to \infty$

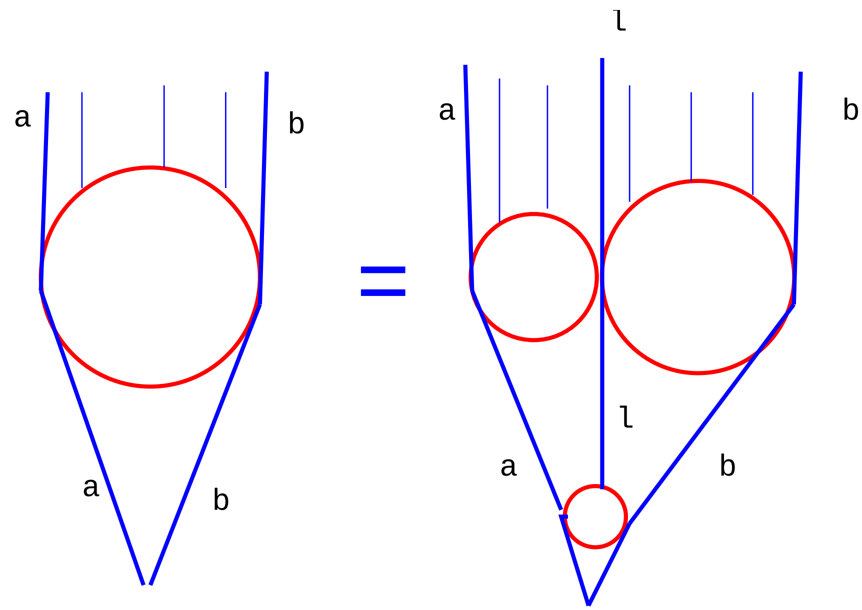
No hard quark recoil

HERWIG (1984) based on collinear analysis: OK

Attempt of improving: Energy-ordered evolution

$$\omega_{i_n} \ll \cdots \ll \omega_{i_1} \ll E$$

$$W_{ab}(q_1 \cdots q_n) = \frac{(ab)}{(aq_1)\cdots(q_n b)}$$



Use $W_{ab}(q_1 \cdots q_n) = W_{ab}(q_\ell) \cdot W_{a\ell}(q_1 \cdots q_{\ell-1})\, W_{\ell b}(q_{\ell+1} \cdots q_n)$

Derive $E\partial_E G_{ab}(E) = \int \frac{d\Omega}{4\pi} \bar{\alpha}_s \, \hat{W}_{ab}(q) \Big[u(q)\, G_{aq}(E)\, G_{qb}(E) - G_{ab}(E) \Big]$

Virtual correction in the last term

$$\hat{W}_{ab}(q) = \omega^2\, W_{ab}(q) = \frac{1-\cos\theta_{ab}}{(1-\cos\theta_{aq})(1-\cos\theta_{qb})}$$

No recoil included

Proper for Monte Carlo generator. No hard recoil included

$$G_{ab}(E) = S_{ab}(E,Q_0) + \int \frac{d\omega}{\omega}\frac{d\Omega}{4\pi}\,\bar{\alpha}_s \hat{W}_{ab}(q)\frac{S_{ab}(E,Q_0)}{S_{ab}(\omega,Q_0)} \cdot G_{aq}(\omega)\, G_{qb}(\omega)$$

with $\ln S_{ab}(E,Q_0) = -\int \frac{d\omega}{\omega}\frac{d\Omega}{4\pi}\bar{\alpha}_s\, \hat{W}_{ab}(q)\, \theta(q_t - Q_0)$

Then

$$d\mathcal{P} = \frac{d\omega}{\omega}\frac{d\Omega}{4\pi}\,\bar{\alpha}_s \hat{W}_{ab}(q)\frac{S_{ab}(E,Q_0)}{S_{ab}(\omega,Q_0)} = d\left(\frac{S_{ab}(E)}{S_{ab}(\omega)}\right)\frac{\bar{\alpha}_s \frac{d\Omega}{4\pi}\hat{W}_{ab}(q)}{\int \bar{\alpha}_s \frac{d\Omega}{4\pi}\hat{W}_{ab}(q)}$$

Then

$$\int d\mathcal{P} = 1 - S_{ab}(E,Q_0)$$

$S_{ab}(E,Q_0)$ is the probability for not emitting within $[Q_0,E]$

Mean multiplicity analysis and so on

$$N(\xi_{ab}, E, Q_0) = 1 + \left.\frac{\partial\, G_{ab}(E, Q_0)}{\partial u(q)}\right|_{u(q)=1} \qquad \xi = 1 - \cos\theta$$

Derive

$$E\partial_E N(\xi_{ab}, E, Q_0) = \int \frac{d\Omega}{4\pi} \bar{\alpha}_s\, \hat{W}_{ab}(q) \Big[N(\xi_{aq}) + N(\xi_{qb}) - N(\xi_{ab}) \Big]$$

1) Mean multiplicity

$$E\partial_E N(\xi_{ab}, E, Q_0) = \int_0^{\xi_{ab}} \bar{\alpha}_s \frac{d\xi}{\xi} N(\xi, E, Q_0) + \Delta(\xi_{ab}, E, Q_0)$$

Δ negligible in collinear limit. One obtains the known DL result

$$N(Q = E\sqrt{\xi}) \simeq \left(\frac{Q}{Q_0}\right)^{\gamma^{(0)}} \cdot \left(1 - \frac{\pi^2}{12}\bar{\alpha}_s + \cdots\right) \qquad \gamma^{(0)} = \sqrt{2\bar{\alpha}_s}$$

2) Heavy $q\bar{q}$ pair multiplicity at small velocity

A.Mueller and GM Phys.Lett. (2004); E.Onofri and GM JHEP (2004)

$$\frac{1}{\bar{\alpha}_s} E\partial_E N(\xi_{ab}, E, Q_0) = \int_0^1 \frac{d\eta}{1-\eta}\Big[\frac{N(\eta\xi_{ab})}{\eta} - N(\xi_{ab})\Big] + \int_{\xi_{ab}}^1 \frac{d\eta}{1-\eta}\Big[N(\eta^{-1}\xi_{ab}) - N(\xi_{ab})\Big]$$

BFKL behaviour for ξ_{ab} small:

$$N(\xi_{ab}, E, Q_0) \sim \frac{e^{-(\alpha_P - 1)Y}}{\sqrt{Y}} e^{-\frac{\ln^2(\xi_{ab}^2)}{14 b \bar{\alpha}_s \zeta_3 Y}}$$

3) Away from jet energy flow

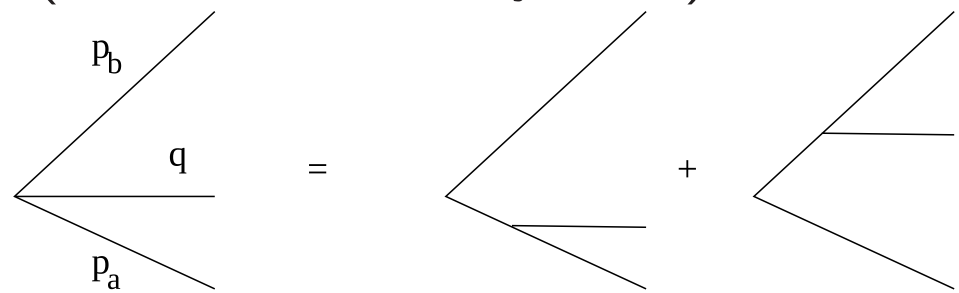
Dasgupta, Salam JHEP (2002); Banfi, Smye GM JHEP JHEP (2003); Appleby, Seymour JHEP (2002); Berger, Kucs, Sterman Phys.Rev. (2003); Dokshitzer GM JHEP (2004)

Collinear factorization: fragmentation formula $D(E,x)$

Consider the inclusive process $e+e^- \to p_a + X$ for soft emission

Recoil of hard partons here needed (follow Catani Seymour)

$$W_{ab}(q) = W_{ab}^{(a)}(q) + W_b^{(b)}(q)$$



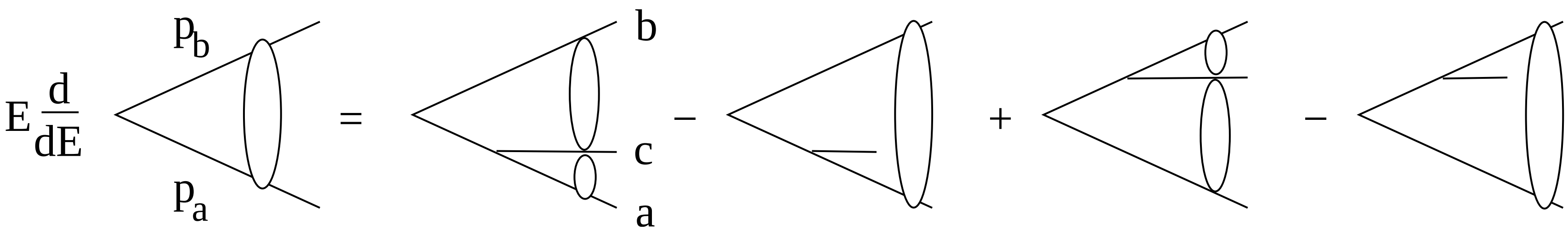
$$\int \frac{d\phi_{aq}}{2\pi} W_{ab}^{(a)}(q) = \frac{1}{\omega^2 \xi_{aq}} \theta(\xi_{ab} - \xi_{aq}) \qquad \xi = 1 - \cos\theta$$

$$W_{ab}^{(a)}(q):\; P_a + P_b \to p_a^{(a)} + p_b^{(a)} + q \qquad \begin{cases} p_b^{(a)} = (1-y)P_b & \text{minimal recoil} \\ p_a^{(a)} = zP_a + (1-z)yP_b - q_t \\ q = (1-z)P_a + zyP_b + q_t \end{cases}$$

$q_t \cdot P_{a,b} = 0.$ Collinear limit $y \to 0$. Infrared limit $y \to 0$ and $z \to 1$

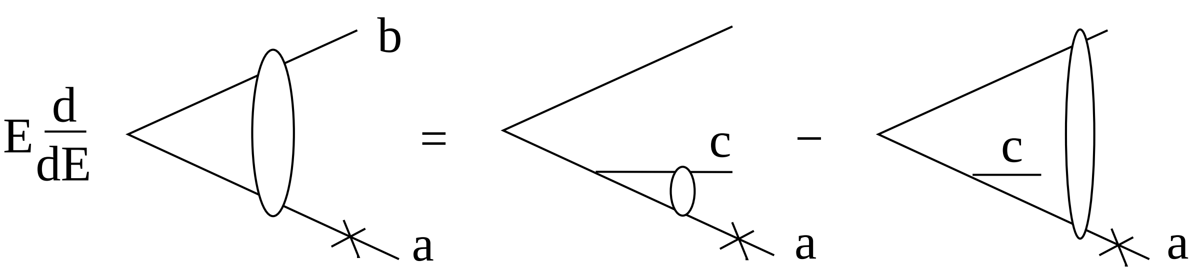
Modify evolution equation with recoil

$$E\partial_E G(p_a\, p_b) = \sum_{c=a,b} \int \frac{d\Omega}{4\pi} \bar{\alpha}_s\, \hat{W}_{ab}^{(c)}(q) \Big[G\left(p_a^{(c)} q\right)\, G\left(q p_b^{(c)}\right) - G(P_a P_b) \Big]$$



Modify equation for structure function $D(E,x)$:

NS structure function for $e^+e^- \to p_a + X$ with $p_a = xP_A$ close to P_a



$\omega = yE < E$

$\xi_{ac} < \xi_{ab}$ ph.sp.integr.

$$D(E\sqrt{\xi_{ab}}, x) \qquad \left[D\left(\omega\sqrt{\xi_{ac}}, \frac{x}{1-y}\right) - D(\omega\sqrt{\xi_{ab}}, x) \right]$$

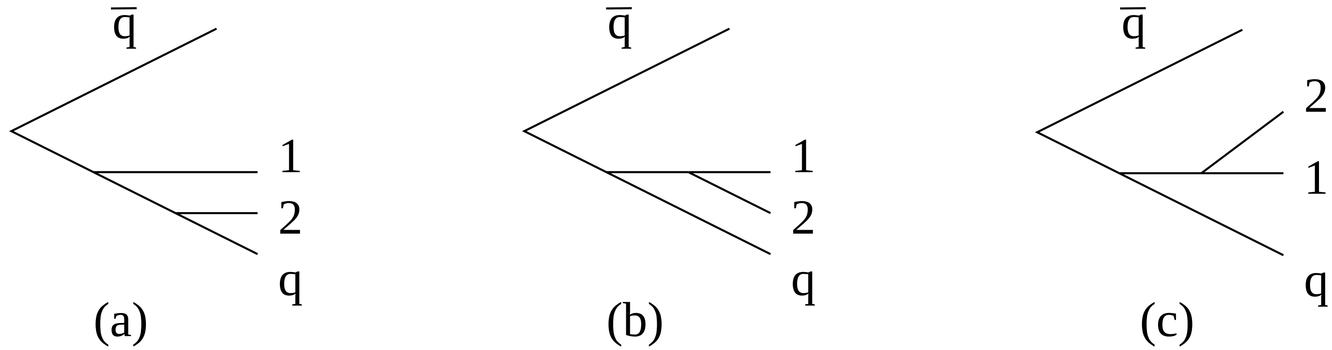
$$D(E\sqrt{\xi_{ab}},x) = \int^{\xi_{ab}} \frac{d\xi_{ac}}{2\xi_{ac}} \int^1 \frac{dy}{y} \bar{\alpha}_s \left[D\left(yE\sqrt{\xi_{ac}}, \frac{x}{1-y} \right) - D(yE\sqrt{\xi_{ab}},x) \right]$$

No DGLAP equation starting at two loops

DGLAP equation: using $Q = E\sqrt{\xi_{ab}}$ and $q_t = (1-z)E\sqrt{\xi_{ac}}$

$$D(Q,x) = \int^{Q^2} \frac{dq_t^2}{2q_t^2} \int^1 \frac{dz}{1-z} \bar{\alpha}_s \left[D\left(q_t, \frac{x}{z} \right) - D(q_t,x) \right]$$

Two loop:



$\theta(\xi_{q1} - \xi_{q2})$ $\theta(\xi_{q1} - \xi_{12})$ $\theta(\xi_{1\bar{q}} - \xi_{12})$

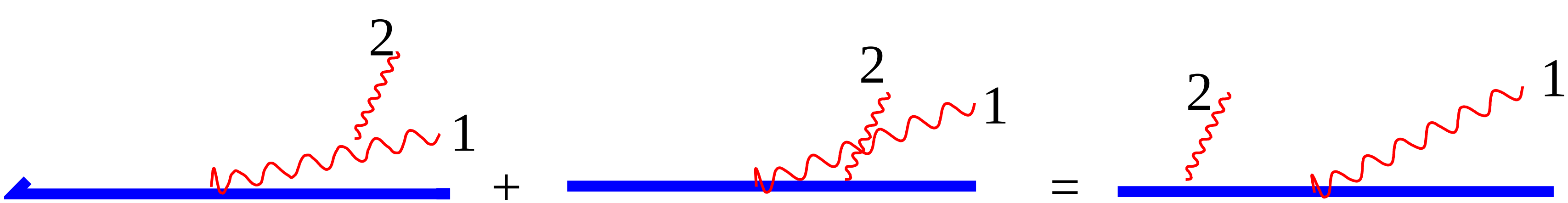
Graph (b) and (c) canceled by virtual corrections

Only graph (a) and its virtual correction survives

Final phase space: $\omega_2 < \omega_1 < E$ kinematical ordering

$\xi_{q2} < \xi_{q1} < \xi_{q\bar{q}}$ phase space integration

No DGLAP equation. Coherence is lost



Use energy ordering

$$\bar{\alpha}_s^2 \int_0^E \frac{d\omega_1}{\omega_1} \int_0^{\omega_1} \frac{d\omega_2}{\omega_2} \Omega(\omega_1\omega_2, x) \int_{Q_0}^{Q} \frac{dq_{t1}}{q_{t1}} \int_{Q_0}^{Q} \frac{dq_{t2}}{q_{t2}} \cdot \theta\left(\frac{q_{t1}^2}{\omega_1^2} - \frac{q_{t2}^2}{\omega_2^2}\right)$$

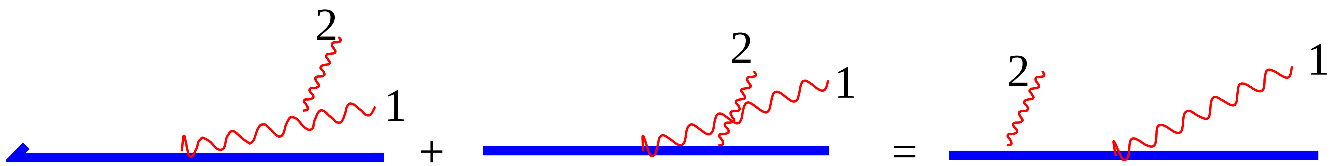
with $\theta\left(\frac{q_{t1}^2}{\omega_1^2} - \frac{q_{t2}^2}{\omega_2^2}\right) \simeq \theta(\cos_{a1} - \cos_{a2})$

But coherence is lost: $\frac{1}{2}\bar{\alpha}_s \ln N \frac{1}{2} \ln^2 \frac{Q}{Q_0} - \frac{1}{6}\bar{\alpha}_s^3 \ln \frac{Q}{Q_0} \ln^3 N$

Use transverse momentum ordering $q_{t1}^2 = \omega_1^2 \sin^2\theta_{a1}$

$$\bar{\alpha}_s^2 \int_0^E \frac{d\omega_1}{\omega_1} \int_0^E \frac{d\omega_2}{\omega_2} \Omega(\omega_1\omega_2, x) \int_{Q_0}^{Q} \frac{dq_{t1}}{q_{t1}} \int_{Q_0}^{q_{t1}} \frac{dq_{t2}}{q_{t2}} \cdot \theta\left(\frac{q_{t1}^2}{\omega_1^2} - \frac{q_{t2}^2}{\omega_2^2}\right)$$

with $\theta\left(\frac{q_{t1}^2}{\omega_1^2} - \frac{q_{t2}^2}{\omega_2^2}\right) \simeq \theta(\cos_{a1} - \cos_{a2})$. But coherence is lost



$$\int_{Q_0}^{Q} \frac{dq_{t1}}{q_{t1}} \int_{Q_0}^{q_{t1}} \frac{dq_{t2}}{q_{t2}} \cdot \theta\left(\frac{q_{t1}^2}{\omega_1^2} - \frac{q_{t2}^2}{\omega_2^2}\right) = \frac{1}{2}\ln^2\frac{E}{Q_0} + \ln\frac{E}{Q_0}\ln\frac{\omega_2}{\omega_1}\theta(\omega_1 - \omega_2)$$

Then $\gamma_N = \bar{\alpha}_s \ln N - \frac{1}{6}\bar{\alpha}_s^2 \ln^3 N + \cdots$

The anomalous dimension is wrong for a contribution at two loops

Conclusion on Monte Carlo for dipole emission

1) Monte Carlo for dipole emission with energy ordering:

convenient for multiplicity averages including large angle emission

necessary for non-global logs observables (Dasgupta and Salam 2001)

2) Including recoil in a simple way:

no DGLAP evolution: Monte Carlo is impossible

3) Resummation of collinear logs is still fine